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Elliptical Water Jets

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Introduction

STUDIES of elliptical water jets have a very considerable history. In 1879 Lord Rayleigh[1] described tests on water jets discharging from elliptical orifices, and referred to earlier tests made "many years ago" by Bidone. Both Bidone and Rayleigh noticed the rather remarkable change with axial distance in the cross section of jets from elliptical orifices. If the major axis of the ellipse is, say, in the horizontal plane at the nozzle, in a short distance downstream the jet has changed so that its major axis is vertical.

This change in jet shape has been nicely sketched by Sir Geoffrey Taylor,[2] who also studied elliptic jets, and an adaptation of Taylor's sketch is given in Fig. 1. In the absence of surface tension, as in a gaseous jet, this is the final geometry of the jet, but, if surface tension is present, additional cycles are possible, the jet alternately switching major and minor axis cross sections.

Both Rayleigh and Taylor suggest that the change of shape of elliptic jets is due to the flow from the outer portions of the ellipse (on the major axis) converging together in the manner of two circular jets impinging at a shallow angle; the resulting flatness at 90 deg to the ellipse major axis being due to the impingement. Indeed, Taylor[2] used an elliptical orifice flow to model the impinging circular-jet flow situation.

Applications of elliptic jets are not common, but they are occasionally found in heating and ventilating outlets and in eductors. Nonaxisymmetric nozzles are sometimes employed in high-performance aircraft, because of the noise reduction aspects attributed to the "flat" sides of the jet (as discussed by Crighton[3]) or because of better geometrical adaptation to the complete aircraft design.[4] Rectangular nozzles of the type occasionally installed in aircraft produce elliptical-shaped jets complete with 90-deg major axis shift. The present study was motivated by the desire to study elliptic jet flows as an interesting and little understood class of fluid phenomena, and by the hope that using high-speed photography, any changes

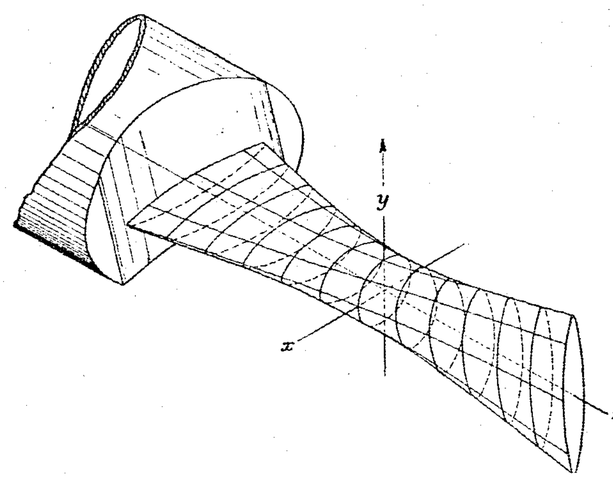
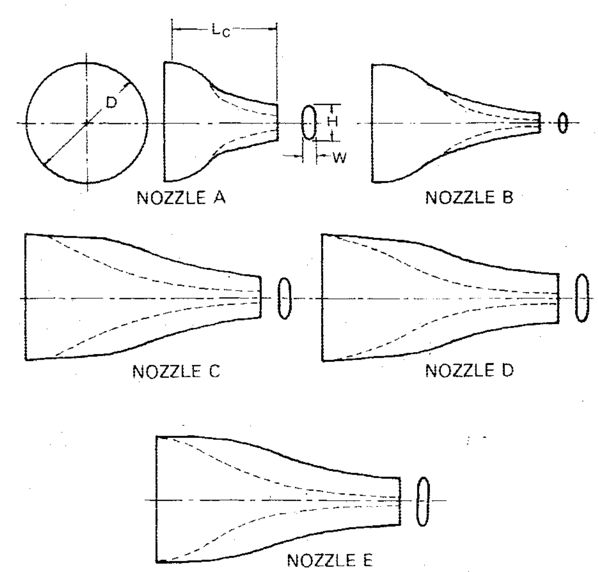


Fig. 1 Diagram of jet issuing from an elliptic orifice (after Taylor[2]).



NOZZLE DIMENSIONS, INCHES

NOZZLE	W	H	H/W	Lc	D	CONTRACTION RATIO
A	0.215	0.545	2.53	1.6	1.85	28.9
B	0.117	0.312	2.67	2.3	1.75	83.8
C	0.185	0.705	3.81	4.0	2.15	35.5
D	0.174	0.719	4.13	3.5	1.90	28.8
E	0.150	0.718	4.79	3.6	1.80	30.0

Fig. 2 Sketch of internal dimensions of nozzles used in the study (dashed lines show projections perpendicular to minor axis of ellipse).

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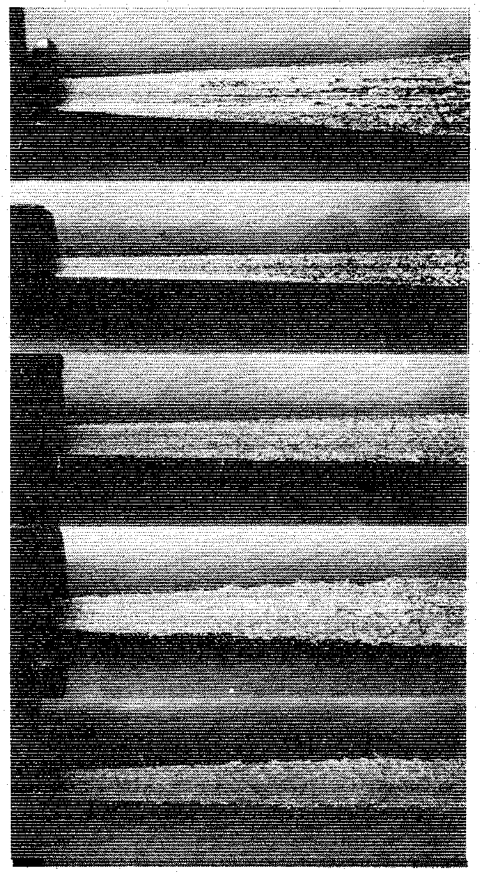

Fig. 3 Water discharge from nozzle A to E (top to bottom), with major axis of nozzles horizontal. Nozzle base pressure: 50 psi.

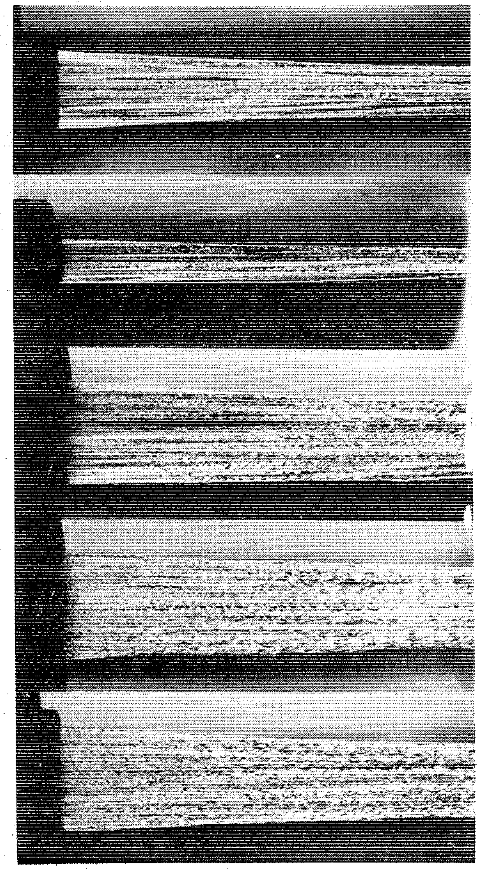

Fig. 4 Polymer solution discharge from nozzles A to E (top to bottom), with major axis of nozzle vertical. Nozzle base pressure: 50 psi.

in jet stability due to the jet eccentricity could be made visible, thus providing information relative to the claim of lower noise radiation from the "flat" sides of elliptic jets.

Test Setup

The nozzles used in this study were cast in white metal from plaster molds, using the "lost wax" process. A sketch of the principal nozzle internal dimensions is given in Fig. 2, which also shows some of the pertinent nozzle data. The five nozzles used in the study, designated A to E, cover a range of nozzle contraction ratios, axial contraction lengths, and nozzle eccentricities.

Water or premixed polymer solutions could be supplied from a 650-gal pressure tank. A nozzle base pressure of 50 psi was used for all tests. A honeycomb flow straightener was placed in the inlet piping of the nozzle. The nozzle discharge was photographed in the plane of each axis by rotating the nozzle 90 deg so that the major axis of the elliptical discharge was either vertical or horizontal. Photography was accomplished using a specially designed camera (Taylor[5]) which achieves good definition by moving the cut film so as to provide image-motion compensation for the predominantly axial motion of the jet. The actual exposure is made by 8-μs electronic flashes, arranged to illuminate the jet at 45 deg above and below the camera axis. The jets are shielded from ambient light by a light-trap arrangement.

In addition to tests with water, additional runs were made using a 200 part-per-million solution of poly (ethylene oxide), a drag reducing polymer having a nominal molecular weight of 4×10^6. Tests with polymer solutions instead of water are useful to indicate the flow phenomena since the viscoelastic properties of the polymer solution inhibit the small-scale instabilities and spray formation activities of the pure water jet, without observable effect on the large-scale instabilities. Based on the viscosity of water, all tests were made at a nominal jet exit Reynolds number of 2×10^5, i.e., fully turbulent flow.

Results and Discussion

Figure 3 shows the discharge from the five nozzles with the major axis of the nozzle horizontal, and indicates how the flow in the minor-axis plane expands, in good accordance with G.I. Taylor's sketch (Fig. 1). All of the photographs have been printed upside down so as to show the flow from left to right. Figure 4 shows the same nozzles, photographed with the major axis of the nozzle vertical. From this aspect, the emerging flow appears to contract. Again, this appearance is in accordance with the sketch of Fig. 1.

Examining Figs. 3 and 4, regions in each nozzle flow having the same projected width in both horizontal and vertical photographs should have an approximate axisymmetric cross section. Figure 5 shows that the location of the initial circular section is a linear function of the nozzle eccentricity, for constant nozzle inlet pressure.

Sforza et al.[6] suggest that initially nonaxisymmetric jets tend to axisymmetric flow far downstream. Photographs taken further downstream do not support this conclusion and nozzle B, for example shows a marked asymmetry at 308 minor axis diameters downstream. So far as jet breakup itself is concerned, elliptic jets seem about as coherent as jets from round nozzles, both tending to break up in stagnant air at about 240 nozzle diameters downstream. While circular water jets break up due to helical instabilities,[7] no such regular breakup was found with the elliptical nozzles.

With polymer solution flow, the minor surface instabilities and spray detachment modes are suppressed, and the basic form of the jet flow becomes clearer, as illustrated by comparing Figs. 3 and 4.

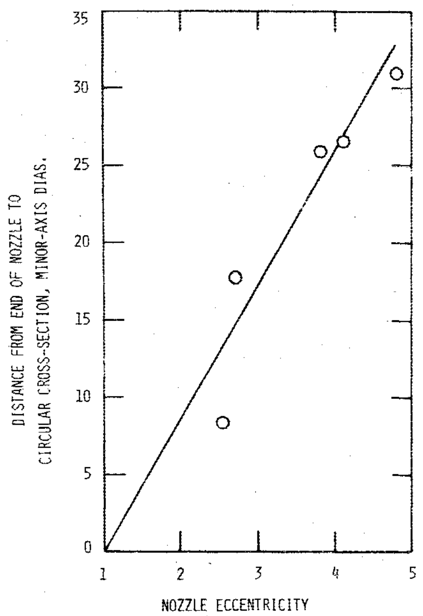

Fig. 5 Distance from nozzle exit (in minor axis diameters) to initial circular cross section of jet, as a function of nozzle eccentricity *e*.

Now we examine the claim, mentioned by Crighton,[3] that the "flat" part of an elliptical aircraft nozzle jet radiates less noise than a similar circular jet. We can see no instability patterns, or lack of them, on the surfaces of the jet to explain a lower radiation from the wide part of the jet, and indeed Maestrello and McDaid[8] found very little difference in near-field noise emission from various sides of a rectangular jet. What could be happening, however, is the 90-deg shift in major axis shortly after leaving the aircraft, which then presents a smaller width radiating source to observers on the ground than would a circular jet of the same power. This then could explain the possible noise benefits of elliptical nozzles, although more elaborate theories have been proposed.[9]

Acknowledgment

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Pseudo-transonic Equation with a Diffusion Term

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Introduction

THE purpose of this Note is to show how Hayes'[1] pseudo-transonic equation for supersonic flow, when modified by a streamwise diffusion term, can be transformed into Burgers' equation. The solution of this equation reduces to that of Whitham[2] in the limit of vanishing viscosity. The role of diffusion near shocks is well known, and in this Note certain simplifying features introduced by viscosity are discussed.

Analysis

Ackeret's classical solution is based on an approximate linearized equation that is a proper first approximation near the surface, but which breaks down at large distances from the airfoil. For example, classical theory fails to predict the bending of Mach lines that occurs in reality, and the subsequent shock formation and decay. This failure arises from the neglected cumulative effects of locally small disturbances that grow to first order over large distances. It is now well known that the nonuniformity of linear theory arises only from the neglect of the "pseudo-transonic" term $(\gamma+1)M_\infty^4 \varphi_x\varphi_{xx}$ in the equation for the disturbance velocity potential φ, where γ is the ratio of specific heats, M_∞ is the freestream Mach number, x is the streamwise coordinate and y is to be the stream-normal coordinate; the contribution of all other nonlinear terms is uniformly of second order. This pseudo-transonic term has a first-order cumulative effect and must be retained in addition to the usual linear ones in seeking a uniformly valid first approximation. To the same order of accuracy, the tangency condition can be imposed on the axis, and this completes the usual nonlinear, supersonic, *inviscid* formulation.[1,2]

In this Note, the physical problem is considered as a limit of the diffusive system

$$\delta\varphi_{xxx} + (1 - M_\infty^2 - (\gamma+1)M_\infty^4\varphi_x)\varphi_{xx} + \varphi_{yy}(x,y) = 0 \qquad (1a)$$

$$\varphi_y(x,0) = \epsilon T'(x) \qquad (1b)$$

$$\varphi = 0 \text{ upstream} \qquad (1c)$$

where $\delta > 0$ is a small diffusion coefficient, ϵ is the thickness ratio, and T' is a normalized slope. It is interesting that Eq. (1) is just the "viscous transonic equation" as derived by Cole,[3] although in the present application we are dealing with purely supersonic applications. In the transonic case, Eq. (1) is obtained through a special limiting process taking into account the effect of compressive viscosity at sonic lines and shocks, where the coefficient of φ_{xx} normally would vanish in low-order theory; to the order considered, the effects of rotationality introduced by viscosity and shock curvature can be ignored. We can expect solutions of Eq. (1) to reduce in the limit of $\delta \to 0$ to solutions of the inviscid formulation; that they actually do in the transonic case was demonstrated by

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